

Restriction on the strong coupling constant in the IR region from the 1D-1P splitting in bottomonium

A. M. Badalian and A. I. Veselov

Institute of Theoretical and Experimental Physics, B. Cheremushkinskaya 25, 117218 Moscow, Russia

B. L. G. Bakker

Vrije Universiteit, Department of Physics, Amsterdam, The Netherlands

(Received 1 November 2003; revised manuscript received 9 February 2004; published 30 July 2004)

The $b\bar{b}$ spectrum is calculated with the use of a relativistic Hamiltonian where the gluon-exchange potential between a quark and an antiquark is taken as in background perturbation theory. We observed that the splittings $\Delta_1 = Y(1D) - \chi_b(1P)$ and other splittings between low-lying states are very sensitive to the QCD constant $\Lambda_V(n_f)$ which occurs in the vector scheme, and good agreement with the experimental data is obtained for $\Lambda_V(2\text{-loop}, n_f=5) = 325 \pm 10$ MeV which corresponds to the conventional $\Lambda_{\overline{MS}}(2\text{-loop}, n_f=5) = 238 \pm 7$ MeV, $\alpha_s(2\text{-loop}, M_Z) = 0.1189 \pm 0.0005$, and to a large freezing value of the background coupling: $\alpha_{\text{crit}}(2\text{-loop}, q^2=0) = \alpha_{\text{crit}}(2\text{-loop}, r \rightarrow \infty) = 0.58 \pm 0.02$. If the asymptotic freedom behavior of the coupling is neglected and an effective freezing coupling $\alpha_{\text{static}} = \text{const}$ is introduced, as in the Cornell potential, then precise agreement with $\Delta_1(\text{exp})$ and $\Delta_2(\text{exp})$ can be reached for the rather large Coulomb constant $\alpha_{\text{static}} = 0.43 \pm 0.02$. We predict a value for the mass $M(2D) = 10451 \pm 2$ MeV.

DOI: 10.1103/PhysRevD.70.016007

PACS number(s): 11.15.Tk, 12.38.Lg, 14.40.Gx

I. INTRODUCTION

Recently the CLEO Collaboration has discovered the first stable D-wave state in bottomonium with a mass $M(1D) = 10162.2 \pm 1.6$ MeV which is consistent with the $J=2$ assignment [1]. The observation of this state is also a great success of the potential model (PM) since just in the framework of the PM approach the strategy how to observe the 1D state was developed and correct values for the radiative transitions and the $M(1D)$ mass were obtained [2,3]. A study of the orbital and radial $b\bar{b}$ excitations which have relatively large sizes is also very important for a better understanding of the fundamental interaction in the infrared (IR) region. At present different conceptions about the value of the strong coupling in the IR region exist. In lattice QCD one finds for the coupling constant in the static potential, parametrized as a linear plus Coulomb potential, the small value $\alpha_{\text{static}} = 0.23(n_f=0) - 0.30(n_f=3)$ [4]. In phenomenological potentials the Coulomb constant is larger but spreads over a wide range from $\alpha_{\text{static}} \approx 0.33 - 0.39$ [5,6] up to the rather large values $0.42 - 0.45$ adopted in recent calculations [7,8]. If the asymptotic freedom (AF) behavior is taken into account, then an even larger freezing value of the physical coupling, $\alpha_s(1 \text{ GeV}) \sim 0.9 \pm 0.1$, was determined from the analysis of the hadronic decays of the τ -lepton [9]. Our point of view discussed here is that after the discovery of the $Y(1D)$ the bottomonium spectrum already contains enough information to determine the freezing value of the coupling constant in an unambiguous way.

Among the excited states in bottomonium the most precise information about the static interaction at large r can be extracted from an analysis of the 1P, 2P, 1D, and 2D states, because they have large sizes, lie below the open flavor threshold, and have no hadronic shifts. We shall show that instead of the absolute masses it is more convenient to use

the splittings between *orbital* excitations since they are less sensitive to the choice of such parameters as the b -quark mass m and in the mass differences the relativistic corrections (about $\lesssim 20$ MeV) are partially or completely cancelled. Note that for these states the spin-averaged masses $M_{\text{cog}}(nL)$, and therefore also the splittings between them, are known with great accuracy, ~ 1 MeV. For the nS states [since the states $\eta_b(nS)$ are still unobserved] the accuracy of the spin-averaged masses $M_{\text{cog}}(nS)$ as well as the splittings 2S-1S and 1P-1S are only about 10 MeV.

From experiment we take two splittings between the spin-averaged masses for orbital excitations:

$$\Delta_1 = M_{\text{cog}}(1D) - M_{\text{cog}}(1P) = 262.1 \pm 2.2(\text{exp})_{-0}^{+1}(\text{th}) \text{ MeV},$$

$$\Delta_2 = M_{\text{cog}}(2P) - M_{\text{cog}}(1P) = 360.1 \pm 1.2(\text{exp}) \text{ MeV}, \quad (1.1)$$

which are measured, as well as the masses $M_{\text{cog}}(1P) = 9900.1 \pm 0.6$ MeV, $M_{\text{cog}}(2P) = 10260.0 \pm 0.6$ MeV, and $M(1D_2) = 10162.2 \pm 1.6$ MeV with high accuracy. However, for $M_{\text{cog}}(1D)$ one needs to take into account the very small difference between $M_{\text{cog}}(1D)$ and $M(1D_2)$ coming from the fine structure splittings: $M_{\text{cog}}(1D) = M_{\text{exp}}(1D_2) + \delta_{\text{FS}}$. In Ref. [3] δ_{FS} was found to be small, lying in the range $0 \leq \delta_{\text{FS}} \leq 1$ MeV so that δ_{FS} was included in the theoretical error in Eq. (1.1).

In this paper we show that the splitting Δ_1 weakly depends on the kinematics, the string tension σ and the pole mass of the b quark, being at the same time very sensitive to the gluon-exchange potential used. Therefore Δ_1 [as well as $\Delta_3 = M(2D) - M(2P)$] can be used as a probe of the freezing value of the strong coupling in the IR region.

II. HAMILTONIAN FOR SPINLESS QUARK AND ANTIQUARK

In calculations of the $b\bar{b}$ spectrum we use the relativistic (string) Hamiltonian H_R which is derived from the Fock–Feynman–Schwinger (FFS) representation of the meson Green’s function in QCD [10–12]. This Hamiltonian and the calculated meson masses $M(nL)$ contain only fundamental quantities: the current (pole) mass of the b quark, the string tension σ , and the QCD constant $\Lambda_{\overline{\text{MS}}}(n_f)$. It is known that in QCD, besides the current quark masses, only one “external” mass scale must occur, say the QCD constant $\Lambda_{\overline{\text{MS}}}$, while the string tension has to be expressed through this scale. Such a relation is established in lattice QCD; however, in analytical approaches, including BPT, a relation between σ and Λ is still not derived.

The Hamiltonian H_R is derived using several approximations.

(i) First, the $b\bar{b}$ states below the open beauty threshold are described in the one-channel approximation, i.e., the creation of an additional quark-antiquark pair is neglected.

(ii) Second, the spin-dependent terms in the $Q\bar{Q}$ interaction are considered as a perturbation.

(iii) Third, the minimal area law is used for the vacuum average of the Wilson loop. Then the Hamiltonian H_R has the following simple form:

$$H_R = \omega + \frac{m_q^2}{\omega} + \frac{\mathbf{p}^2}{\omega} + V_{\text{st}}(r), \quad (2.1)$$

where $\mathbf{p}^2 = \mathbf{p}_r^2 + \mathbf{L}^2/r^2$ and the “einbein” variable ω appears when in the Green’s function in the FFS representation one goes over from the proper time τ to the actual time $t: 2\omega = dt/d\tau$. According to the quantization procedure the operator ω is determined from the extremum condition [10,13]:

$$\frac{\partial H_R}{\partial \omega} = 0 \Rightarrow \omega = \sqrt{\mathbf{p}^2 + m_q^2}. \quad (2.2)$$

Therefore the Hamiltonian (2.1) reduces to an expression which does not explicitly depend on ω ,

$$H_R = 2\sqrt{\mathbf{p}^2 + m_q^2} + V_{\text{st}}(r). \quad (2.3)$$

The kinetic term in Eq. (2.3) was postulated many years ago [14] and successfully used in the relativized potential model [6,7,13–16], however, the derivation of this term directly from the QCD meson Green’s function was done for the first time in Ref. [10]. The Hamiltonian (2.1) or (2.3) also implies a definite prescription for m_q while in the PM the quark mass m_q is usually considered as a fitting parameter.

Let us consider two limiting cases. The first one occurs when the nonperturbative (NP) contribution dominates in the $Q\bar{Q}$ potential (at relatively large distances) and the $Q\bar{Q}$ gluon-exchange term for excited states can be considered as a perturbation [13]. Such an approximation is valid for light mesons ($m_q = 0$). Then by derivation the mass m_q in Hamiltonian (2.1) coincides with the Lagrangian current mass (the

difference between current and pole masses can be neglected in the approximation considered):

$$m_q = \bar{m}_q(\bar{m}_q), \quad V(r) = V_{\text{st}}^{\text{NP}}(r) = \sigma r. \quad (2.4)$$

One must also take into account the NP self-energy correction to the quark masses [17]; however, in bottomonium the NP self-energy contribution to H_R appears to be compatible with zero (~ 3 MeV) and can be neglected [18]. For the linear potential the splitting Δ_1 between the orbital excitations 1D and 1P turns out to be ~ 160 – 170 MeV, i.e., much smaller than the experimental value (1.1).

A different situation takes place at small distances where the perturbative $Q\bar{Q}$ interaction dominates while in first approximation the string interaction can be neglected. Then due to perturbative self-energy corrections the pole mass m_q appears in the Hamiltonian (2.1) and for a consistent description $m_q(\text{pole})$ has to be taken in the same n -loop approximation as the perturbative $Q\bar{Q}$ potential:

$$m_q = m_q^{\text{pole}}(n\text{-loop}), \quad V_{\text{st}}^{\text{P}}(r) = -\frac{4}{3} \frac{\alpha_V(n\text{-loop})}{r}. \quad (2.5)$$

For low-lying $b\bar{b}$ states this type of calculations was performed in Ref. [19] [with the use also of the $1/m_q$ expansion of the kinetic term in Eq. (2.3)].

The perturbative static potential in Eq. (2.5) contains the vector coupling $\alpha_V(r)$ in coordinate space. Its relation to the vector coupling $\alpha_V(q)$ in momentum space and to the QCD strong coupling constant $\alpha_s(q)$ in the $\overline{\text{MS}}$ scheme (up to three-loops) was studied in detail in Ref. [20] where the constant $\Lambda_V(\Lambda_R)$, which determines the perturbative vector coupling α_V in momentum (coordinate) space, was expressed through the QCD constant $\Lambda_{\overline{\text{MS}}}$:

$$\Lambda_V(n_f) = \Lambda_{\overline{\text{MS}}}(n_f) \exp\left(\frac{a_1}{2\beta_0}\right), \quad \Lambda_R(n_f) = \Lambda_V^{\gamma_E}, \quad (2.6)$$

where $a_1 = \frac{31}{3} - \frac{10}{9}n_f$, $\beta_0 = 11 - \frac{2}{3}n_f$, and γ_E is the Euler constant.

If one takes the conventional value $\Lambda_{\overline{\text{MS}}}^{(5)}(2\text{-loop}) = 216 \pm 25$ MeV, which corresponds to the “world average” $\alpha_s(2\text{-loop}, M_Z) = 0.1172 \pm 0.0020$ [21], then from the definitions Eq. (2.6) it follows that

$$\Lambda_V^{(5)}(2\text{-loop}) = 295 \pm 34 \text{ MeV}, \quad \Lambda_R^{(5)}(2\text{-loop}) = 525 \pm 61 \text{ MeV}. \quad (2.7)$$

Thus both $\Lambda_V^{(5)}$ and $\Lambda_R^{(5)}$ appear to be essentially larger than $\Lambda_{\overline{\text{MS}}}^{(5)}$ and therefore the perturbative description (where $\Lambda_R r \ll 1$ or $r \ll 0.3$ fm), in a strict sense, can be applied only to the ground state $\Upsilon(1S)$. (The validity of the perturbative static potential in quenched approximation was discussed in Refs. [22,23].) Since the orbital and radial excitations in bottomonium have rather large sizes, e.g., $R(1P) = 0.4$ fm,

$R(1D)=0.5$ fm, $R(2P)=0.6$ fm, and $R(2D)=0.7$ fm, for them the $Q\bar{Q}$ gluon exchange cannot be described in the framework of PQCD alone and a NP modification of $\alpha_s(\alpha_V)$ at large distances (IR freezing) should be taken into account. In our calculations we shall use $\Lambda_V^{(5)}$ from the range (2.7).

III. STATIC POTENTIAL

To describe the $b\bar{b}$ spectrum as a whole we shall use here the static $Q\bar{Q}$ potential as it is defined in background perturbation theory (BPT), where the influence of the background field on the gluon exchange is taken into account [11,24]:

$$V_{\text{st}} = \sigma r + V_B(r), \quad (3.1)$$

$$V_B(r) = -\frac{4}{3} \frac{\alpha_B(r)}{r}. \quad (3.2)$$

Here the background coupling $\alpha_B(r)$ is defined through the Fourier transform of the potential $V_B(q)$ in momentum space [23]; it gives

$$\alpha_B(r) = \frac{2}{\pi} \int_0^\infty \frac{dq}{q} \sin(qr) \alpha_B(q), \quad (3.3)$$

where the background coupling in momentum space $\alpha_B(q)$ is defined over the whole region $0 \leq q < \infty$. For example, in two-loop approximation

$$\alpha_B(q) = \frac{4\pi}{\beta_0} \frac{1}{t_B} \left(1 - \frac{\beta_1}{\beta_0^2} \frac{\ln t_B}{t_B} \right), \quad (3.4)$$

$$t_B = \ln \frac{q^2 + M_B^2}{\Lambda_V^2},$$

where the background mass M_B under the logarithm is determined by the lowest hybrid excitation of the string. This mass cannot be considered as an additional (fitting) parameter since M_B itself can be calculated either on the lattice or with the use of the corresponding Hamiltonian for a hybrid [12]. We take here the value of M_B which was obtained in Ref. [23] from a fit to the static potential on the lattice at small distances [22],

$$M_B = 1.00 \pm 0.05 \text{ GeV}. \quad (3.5)$$

Note that the most transparent way suggested to derive the expression (3.4) for $\alpha_B(q)$ is to consider a large number of colors, $N_c \gg 1$ [11].

The background coupling in momentum space $\alpha_B(q)$ has an important feature—the correct PQCD limit at $q^2 \gg M_B^2$ (as in Ref. [20]) and therefore the constant Λ_V in the vector scheme (3.4) used in our calculations is determined by the conventional $\Lambda_{\overline{\text{MS}}}(n_f)$ according to the relation (2.6).

One should keep in mind that the additive form of the static potential (3.1) is automatically obtained in the framework of BPT in the lowest approximation when the interference terms are neglected. This form is well supported by the

lattice data [25] where the static potential is taken as the sum of a Coulomb-type term plus linear term at separations $r \geq 0.2$ fm. Note that both potentials satisfy the Casimir scaling law with an accuracy of a few percent [26].

In the next approximation of BPT the interference between perturbative and NP effects appears in the form of the background mass M_B (3.4) which ensures the IR freezing of $\alpha_B(q)$. One can see from Eq. (3.4) that the only effect of M_B is to moderate the IR behavior of the perturbative potential, whereby the Landau ghost pole and IR renormalons disappear, while the short distance behavior, as well as the Casimir scaling property, stay intact [11]. Note that the logarithm in the coupling (3.4) formally coincides with that suggested in Ref. [27] with $M_B^2 = 4m_g^2$ where a picture with the gluon acquiring an effective mass m_g was suggested. However, since a physical gluon has no mass, the parameter $(2m_g)^2$ needs to be reinterpreted in the correct way.

Our calculations of the $b\bar{b}$ spectrum are done for the number of flavors $n_f=5$ when the QCD constant $\Lambda_{\overline{\text{MS}}}^{(5)}$ is well known from high energy processes, $\Lambda_{\overline{\text{MS}}}^{(5)} = 216 \pm 25$ MeV, which corresponds to $\alpha_s(M_z) = 0.117 \pm 0.002$ [20] and correspondingly in the vector scheme $\Lambda_V^{(5)} = 295 \pm 34$ MeV. Later in our calculations we show that the splittings Δ_1 and Δ_2 appear to be in agreement with experiment only for those values of $\Lambda_V^{(5)}$ (2-loop) which are close to the upper limit in Eq. (2.7),

$$\Lambda_V^{(5)}(2\text{-loop}) = 325 \pm 10 \text{ MeV}, \quad (3.6)$$

which corresponds to

$$\Lambda_{\overline{\text{MS}}}^{(5)}(2\text{-loop}) = 238 \pm 7 \text{ MeV}, \quad \alpha_s(M_z) = 0.1189 \pm 0.0005. \quad (3.7)$$

From the definition (3.4) and for $\Lambda_V^{(5)}$ (3.6) the freezing (critical) value of the background coupling $\alpha_B(q=0) = \alpha_B(r \rightarrow \infty)$ is expected to be

$$\alpha_{\text{crit}}(n_f=5) = 0.56 \pm 0.01 \quad (M_B = 1.0 \text{ GeV}),$$

$$\alpha_{\text{crit}}(n_f=5) = 0.59 \pm 0.01 \quad (M_B = 0.95 \text{ GeV}). \quad (3.8)$$

Notice that from Eq. (3.3) α_{crit} turns out to be the same in momentum and coordinate space.

For a value of $\Lambda_V^{(5)}$ close to the lower bound the freezing value α_{crit} [Eq. (2.7)] turns out to be essentially smaller, e.g., for $\Lambda_{\overline{\text{MS}}}^{(5)} = 190$ MeV ($\Lambda_V^{(5)} = 260$ MeV) the value $\alpha_{\text{crit}} = 0.46$ is obtained. For such a choice of $\Lambda_V^{(5)}$ agreement with experiment for the $b\bar{b}$ spectrum cannot be reached.

For $\Lambda_{\overline{\text{MS}}}^{(5)}$, Eq. (3.7), the pole mass of the b quark can be determined according to the standard procedure (as in PQCD) and for the conventional value $\bar{m}_b(\bar{m}_b) = 4.23 \pm 0.03$ GeV one obtains in 2-loop approximation

$$m_b(\text{pole}) = 4.82 \pm 0.03 \text{ GeV}. \quad (3.9)$$

Thus all fundamental quantities used in our calculations, $m_b(\text{pole})$, Λ_V and the string tension $\sigma \approx 0.18 \pm 0.02 \text{ GeV}^2$ (taken from the slope of the Regge trajectories for mesons),

are taken in correspondence to QCD; they are determined in the narrow ranges (3.6) and (3.9).

IV. RELATIVISTIC CORRECTIONS

Here for the $b\bar{b}$ spectrum we try to calculate the splittings Δ_1 and Δ_2 defined in Eq. (1.1) with high accuracy. First of all it is of interest to compare the spin-averaged masses $M(nL)$ in the relativistic case, where $M(nL)$ coincides with the eigenvalue (e.v.) of the spinless Salpeter equation (SSE),

$$(2\sqrt{p^2 + m^2} + V_{\text{st}}(r))\psi(nL) = M(nL)\psi(nL), \quad (4.1)$$

with the e.v. $\tilde{M}(nL)$ in the nonrelativistic (NR) limit which is widely used in bottomonium,

$$\tilde{M}(nL) = 2m + E(nL). \quad (4.2)$$

Here the “excitation energy” $E(nL)$ is the e.v. of the Schrödinger equation with the same static potential as in Eq. (4.1):

$$V_{\text{st}} = \sigma r - \frac{4}{3} \frac{\alpha_B(r)}{r}. \quad (4.3)$$

The mass $m = m_b(\text{pole})$ and $\alpha_B(r)$ will be taken below in 2-loop approximation.

In Sec. V we also present the $b\bar{b}$ spectrum calculated with the Cornell potential which has been successfully used for many years [8]:

$$V_C(r) = \sigma r - \frac{4}{3} \frac{\alpha_{\text{static}}}{r} + C_0, \quad (4.4)$$

with $\alpha_{\text{static}} = \text{const.}$ Since in α_{static} the (AF) behavior is neglected, this coupling can be considered as an “effective freezing” coupling in the gluon-exchange potential. Notice that $\alpha_B(r)$ in BPT appears indeed to be close to a constant (the critical value) already at rather small $Q\bar{Q}$ separations, $r \gtrsim 0.4$ fm; e.g., for $n_f = 5$ the ratio $\alpha_B(r)/\alpha_{\text{crit}}$ is equal to 0.83, 0.88, 0.92, and 0.95, respectively, for r equal 0.4, 0.5, 0.6, and 0.7 fm.

In Table I the spin-averaged $b\bar{b}$ masses below the $B\bar{B}$ threshold, the splittings

$$\Delta_1 = Y(1D) - Y(1P), \quad \Delta_2 = Y(2P) - Y(1P),$$

$$\Delta_3 = Y(2D) - Y(2P), \quad (4.5)$$

and the relativistic corrections $\delta_R = M(nL) - \tilde{M}(nL)$ are presented for the static potential in BPT Eq. (4.3) with a typical set of parameters taken from the ranges (3.6) and (3.9):

$$m = 4.828 \text{ GeV}, \quad \sigma = 0.178 \text{ GeV}^2,$$

$$\Lambda_V^{(5)} = 330 \text{ MeV},$$

$$M_B = 1.0 \text{ GeV}. \quad (4.6)$$

TABLE I. The spin-averaged $b\bar{b}$ masses $M(nL)$ for the SSE (4.1), $\tilde{M}(nL)$ (4.2) in the NR case, the splittings Δ_1 , Δ_2 , and Δ_3 (4.5), and the relativistic corrections δ_R (all in MeV) for the static potential in BPT (3.1) with the parameters (4.6).

State	$M(nL)$	$\tilde{M}(nL)$	δ_R
	SSE	NR	
1S	9470	9487	-17
2S	10 022	10 042	-20
3S	10 368	10 393	-24
1P	9900	9909	-9
2P	10 266	10 282	-16
3P	10 555	10 577	-22
1D	10 157	10 166	-9
2D	10 458	10 474	-16
Δ_1	257	257	0
Δ_2	367	374	-7
Δ_3	191	191	0

The accuracy of our numerical calculations is ± 1 MeV for the e.v. of the SSE and ± 0.5 MeV for the Schrödinger equation.

From Table I one can see that the relativistic corrections δ_R give a contribution (~ 10 – 22 MeV) to the absolute values of the $b\bar{b}$ mass, nevertheless the splittings Δ_1 and $\Delta_3 = M(2D) - M(2P)$ for the SSE and in the NR case appear to be equal (within ± 1 MeV). This result is practically independent of the choice of such parameters as σ and m . Therefore Δ_1 can be used as an important criterion to distinguish between different choices of the strong coupling. It can be shown that for the S-wave states δ_R is larger and turns out to be more sensitive to the choice of m and σ .

In the splitting $\Delta_2 = M(2P) - M(1P)$ the relativistic corrections are partly cancelled with Δ_2^{NR} being 7 MeV larger than Δ_2 for the SSE. Therefore, it is better to use the SSE for the calculation of the highly excited $b\bar{b}$ states. (For the Cornell potential we find practically the same values for δ_R .)

TABLE II. The $b\bar{b}$ spin-averaged masses $M(nL)$ (in MeV) for the SSE and the splittings $\Delta_1 = M(1D) - M(1P)$, $\Delta_2 = M(2P) - M(1P)$, and $\Delta_3 = M(2D) - M(2P)$ for different values of m and $\Lambda_V^{(5)}$ (or $\Lambda_{\text{MS}}^{(5)}$) given by Eq. (5.1). The values $\sigma = 0.178 \text{ GeV}^2$ and $M_B = 0.95 \text{ GeV}$ were kept fixed.

State	I	II	III	Experiment
1S	9484	9473	9468	9460.3 ± 0.3 (1^3S_1)
2S	10 023	10 023	10 023	$10 023.3 \pm 0.3$ (2^3S_1)
3S	10 364	10 370	10 372	$10 355.2 \pm 0.5$ (3^3S_1)
1P	9900	9900	9900	9900.1 ± 0.6
2P	10 262	10 266	10 269	$10 260.0 \pm 0.6$
1D	10 152	10 157	10 159	$10 162.2 \pm 1.6$ (exp) $_{-0.0}^{+1.0}$ (th)
2D	10 450	10 457	10 461	...
Δ_1	252	257	259	262.1 ± 2.2 (exp) $_{-0.0}^{+1.0}$ (th)
Δ_2	362	366	369	360.1 ± 1.2
Δ_3	188	190	192	...

TABLE III. The splittings $\Delta_1 = M(1D) - M(1P)$, $\Delta_2 = M(2P) - M(1P)$, $M(2S) - M(1S)$, and $M(1P) - M(1S)$ (in MeV) for $\Lambda_V^{(5)} = 280$ MeV ($\Lambda_{\overline{MS}}^{(5)} = 205$ MeV) and different σ (in GeV^2) ($M_B = 1.0$ GeV, $m_b = 4.82$ GeV).

State	$\sigma = 0.170$	$\sigma = 0.174$	$\sigma = 0.178$	$\sigma = 0.182$	$\sigma = 0.185$	Experiment
Δ_1	241	243	246	251	251	262.1 ± 2.2
Δ_2	347	351	356	360	363	360.1 ± 1.2
$M(1P) - M(1S)$	396	399	402	406	408	$\approx 430^a$
$M(2S) - M(1S)$	514	518	523	529	533	$\approx 550^a$

V. ANALYSIS OF THE SPECTRUM

To demonstrate the sensitivity of the $b\bar{b}$ masses and the splittings to the choice of $\Lambda_V^{(n_f)}$ present in the coupling $\alpha_B(r)$ we give in Table II the e.v.'s $M(nL)$ of the SSE for three different values of $\Lambda_V^{(5)}$ in 2-loop approximation taken from the range (3.6) ($\sigma = 0.178$ GeV^2 , $M_B = 0.95$ GeV). We denote these three sets as

$$\begin{aligned}
 \text{I } m &= 4.819 \text{ GeV}, \quad \Lambda_V^{(5)} = 300 \text{ MeV}, \\
 \Lambda_{\overline{MS}}^{(5)} &= 200 \text{ MeV}, \\
 \text{II } m &= 4.830 \text{ GeV}, \quad \Lambda_V^{(5)} = 320 \text{ MeV}, \\
 \Lambda_{\overline{MS}}^{(5)} &= 234 \text{ MeV}, \\
 \text{III } m &= 4.836 \text{ GeV}, \quad \Lambda_V^{(5)} = 340 \text{ MeV}, \\
 \Lambda_{\overline{MS}}^{(5)} &= 249 \text{ MeV}.
 \end{aligned} \tag{5.1}$$

From Table II one can see that the splitting Δ_1 increases with growing $\Lambda_V^{(5)}$ (or $\Lambda_{\overline{MS}}^{(5)}$) and reaches the experimental value at the value $\Lambda_V^{(5)} \approx 325 \pm 10$ MeV which corresponds to

$$\Lambda_{\overline{MS}}^{(5)}(2\text{-loop}) \approx 238 \pm 7 \text{ MeV}, \quad \alpha_s(m_Z) = 0.1189 \pm 0.0005 \tag{5.2}$$

and gives rise to the critical value ($M_B = 0.95$ GeV)

$$\alpha_{\text{crit}}(n_f = 5) \approx 0.58. \tag{5.3}$$

Thus, the conventional $\Lambda_{\overline{MS}}^{(5)}$ (5.2) [close to the upper limit (2.7)] appears to be at the same time compatible with the large freezing value of the background coupling, giving rise to a good description of the $b\bar{b}$ spectrum and of the splittings

Δ_1 and Δ_2 . Note that the critical value (5.3) is in striking agreement with $\alpha_{\text{crit}} = 0.60$ introduced in Ref. [16] in a phenomenological way.

This important statement does depend neither on the value chosen for σ nor on the quark mass. To illustrate this fact we give in Tables III and IV the splittings Δ_1 and Δ_2 as well as the 1P-1S and 2S-1S splittings for different values of σ and two values of $\Lambda_V(n_f = 5)$: $\Lambda_V^{(5)} = 280$ MeV which corresponds to $\Lambda_{\overline{MS}}^{(5)} = 205$ MeV and $\Lambda_V^{(5)} = 335$ MeV which corresponds to $\Lambda_{\overline{MS}}^{(5)} = 245$ MeV (all Λ 's in 2-loop approximation).

Below we would like also to show the dependence of the splittings on the value of the string tension. We take σ in the range 0.165–0.185 GeV^2 and fix the QCD constant $\Lambda_V^{(5)} = 280$ MeV. [This value of $\Lambda_V^{(5)}$ corresponds to $\Lambda_{\overline{MS}}^{(5)} = 205$ MeV and $\alpha(M_Z) = 0.116$ for $M_B = 1.0$ GeV.] Note that for such a value of $\Lambda_V^{(5)}$ the freezing value, $\alpha_{\text{crit}} = 0.488$, appears to be 20% smaller than that in Eq. (5.3).

From the numbers presented in Table III one can see that the splitting Δ_1 is slightly increasing from the value 241 MeV up to 251 MeV while σ is changing in a wide range, from 0.17 GeV^2 to 0.185 GeV^2 , being still 10–20 MeV smaller than the experimental number given in Eq. (1.1). Also other splittings between low-lying states, e.g., $M(2S) - M(1S)$ and $M(1P) - M(1S)$, appear to be by 30–50 MeV smaller than the experimental numbers for any σ from the range 0.17–0.185 GeV^2 .

Our calculations show that for a reasonable value of $\Lambda_V^{(5)}$ one cannot reach agreement with experiment by variation of the parameter σ only. Larger values of $\Lambda_V^{(5)}$ (around 320 MeV) are needed to get $\Delta_1 \approx 260$ MeV.

The situation appears to be different if a large value $\Lambda_{\overline{MS}}^{(5)}(2\text{-loop}) = 245$ MeV [$\alpha(M_Z) = 0.1194$] is taken (see Table IV). In this case the same three splittings turn out to be in very good agreement with the experimental numbers for $\sigma = 0.18 \pm 0.02$ GeV^2 .

TABLE IV. The same splittings as in Table III for $\Lambda_V^{(5)} = 335$ MeV ($\Lambda_{\overline{MS}}^{(5)}(2\text{-loop}) = 245$ MeV).

State	$\sigma = 0.170$	$\sigma = 0.174$	$\sigma = 0.178$	$\sigma = 0.182$	$\sigma = 0.185$	Experiment
Δ_1	253	255	258	261	263	262.1 ± 2.2
Δ_2	360	364	368	372	376	360.1 ± 1.2
$M(1P) - M(1S)$	423	426	430	433	436	$\approx 430^a$
$M(2S) - M(1S)$	544	549	554	559	563	$\approx 550^a$

TABLE V. The $b\bar{b}$ spin-averaged masses $\tilde{M}(nL)$ (NR case) for the Cornell potential ($m=4.80$ GeV, $\sigma=0.179$ GeV²) with different values of α_{static} and C_0 (the masses and C_0 are given in MeV).

	$\alpha_{\text{st}}=0.3645$ $C_0=-74$	$\alpha_{\text{st}}=0.4071$ $C_0=-32$	$\alpha_{\text{st}}=0.4263$ $C_0=-16$	Experiment
$\tilde{M}(1S) + \delta_{\text{AF}}(1S)^a$	9421	9434	9451	9460.3 ± 0.3 (1^3S_1)
$\delta_{\text{AF}}(1S)$	-34	+37	+79	
$2S^b$	10 000	9995	9993	$10\,023.3 \pm 0.3$ (2^3S_1)
$3S$	10 337	10 344	10 347	$10\,355.2 \pm 0.5$ (3^3S_1)
$1P$	9900 (fit)	9900 (fit)	9900 (fit)	9900.1 ± 0.6
$2P$	10 249	10 259	10 263	$10\,260.0 \pm 0.6$
$1D$	10 139	10 153	10 158	$10\,162.2 \pm 1.6^{+1.0}_{-0.0}$
$2D$	10 430	10 448	10 458	...
Δ_1	239	253	258	$262.1 \pm 2.2(\text{exp})^{+1.0}_{0.0}$ (th)
Δ_2	347	359	363	360.1 ± 1.2
Δ_3	183	189	192.5	...

^aThe AF correction $\delta_{\text{AF}}(1S)$ is defined in Eq. (5.4).

^bThe AF correction is neglected for all states with the exception of the ground state.

While the calculated 1D-1P, 2S-1S, and 1P-1S splittings are in precise agreement with experiment for $\Lambda_V^{(5)} = 320 - 335$ MeV ($\Lambda_{\text{MS}}^{(5)} = 234 - 245$ MeV), the 2P-1P splitting in these cases appears to be ≈ 10 MeV larger than the experimental one. We expect that a small decrease of $M(2P)$ [and also of $M(3S)$] can be reached taking into account the flattening of the confining potential due to additional pair creation.

The sensitivity of the $b\bar{b}$ spectrum to the Coulomb constant in the Cornell potential is illustrated in Table V where three variants with fixed $m=4.80$ GeV and $\sigma=0.179$ GeV² are presented while $\alpha_{\text{static}} = \text{const}$ is varied. From Table V one can see that only for the choice with large Coulomb constant $\alpha_{\text{static}} = 0.41 - 0.43$, good agreement with experiment is obtained for all states with the exception of the ground state and partly of the 2S state.

For the ground state $Y(1S)$ the AF behavior is very important both for the mass and for the wave function at the origin. Therefore one may take into account the difference between α_{static} in the Cornell potential (playing the role of the effective freezing coupling) and $\alpha_B(r)$ considering it as a perturbation. The perturbative treatment gives rise to the mass correction

$$\delta_{\text{AF}}(1S) = \frac{4}{3} \left\langle \frac{\alpha_{\text{static}} - \alpha_B(r)}{r} \right\rangle_{1S}. \quad (5.4)$$

Owing to the AF effect this correction turns out to be positive (see second row in Table V) for $\alpha_{\text{static}} \geq 0.39$. However, a relatively large value for the AF correction, $\delta_{\text{AF}} \approx 40 - 80$ MeV, for the ground state clearly indicates that the AF effect in the gluon-exchange potential is very important and cannot be treated accurately as a perturbation to the Coulomb potential.

The meaning of the quantity α_{static} in the Cornell potential can be understood if one introduces in BPT the effective coupling for a given nL state according to the following relation:

$$\left\langle \frac{\alpha_B(r)}{r} \right\rangle_{nL} = \alpha_{\text{eff}}(nL) \left\langle \frac{1}{r} \right\rangle. \quad (5.5)$$

The values of $\alpha_{\text{eff}}(nL)$ are given in Table VI from which one can see that for the low-lying states the effective coupling in BPT, $\alpha_{\text{eff}} \approx 0.40 - 0.43$, is indeed very close to the Coulomb constant used in the Cornell potential.

From the spectra presented in Table V one can conclude that the splittings Δ_1 and Δ_2 are in precise agreement with experiment for

$$\alpha_{\text{static}} = 0.43 \pm 0.02 \quad (5.6)$$

and this value appears to be 25% lower than the critical value for the background coupling states given in Eq. (5.3) and very close to the effective coupling given in Table VI.

For the best variants from Tables II and V the splitting $\Delta_3 = M(2D) - M(2P)$ is practically unchanged, $\Delta_3 = 191 \pm 2$ MeV and therefore our prediction for the mass of the excited 2D state is $M_{\text{cog}}(2D) \approx M(2D_2) = 10\,451 \pm 2$ MeV.

VI. CONCLUSION

From our analysis of the bottomonium spectrum we observe that the splittings between orbital excitations Δ_1 , Δ_2 , and Δ_3 appear to be very sensitive to the freezing value of the coupling in the gluon-exchange potential. The splitting $\Delta_1 = M(1D) - M(1P)$ is of special importance since it is measured with the great accuracy.

TABLE VI. The effective coupling $\alpha_{\text{eff}}(nL)$ (5.5) for the static potential with the parameters (4.6).

	State						
	1S	2S	3S	1P	2P	1D	2D
$\alpha_{\text{eff}}(nL)$	0.39	0.41	0.43	0.47	0.47	0.50	0.50

For the Cornell potential, when the Coulomb constant α_{static} can be considered as an effective freezing coupling, precise agreement with experiment takes place for $\alpha_{\text{static}} = 0.43 \pm 0.02$.

For the $Q\bar{Q}$ gluon-exchange potential with the strong coupling taken as in BPT, where the freezing value α_{crit} is fully determined by the value of $\Lambda_V(n_f=5)$ in the vector scheme,

good agreement with experiment is obtained only for $\Lambda_V^{(5)} = 325 \pm 10$ MeV which corresponds to the conventional $\Lambda_{\overline{\text{MS}}}^{(5)}$ close to the upper limit: $\Lambda_{\overline{\text{MS}}}^{(5)}(2\text{-loop}) = 238 \pm 7$ MeV and $\alpha_s(M_Z) = 0.119 \pm 0.001$ and at the same time gives rise to a large freezing value, $\alpha_{\text{crit}} \approx 0.58$.

We also predict that the mass of the as yet unobserved 2D state(s) is $M(2D_J) \approx M_{\text{cog}}(2D) = 10451 \pm 2$ MeV.

-
- [1] CLEO Collaboration, S.E. Csorna *et al.*, hep-ex/0207060; H. Mahlke-Krüger, hep-ex/0309015.
 - [2] J.L. Rosner, Phys. Rev. D **67**, 097504 (2003); W. Kwong, J.L. Rosner, and C. Quigg, Annu. Rev. Nucl. Part. Sci. **37**, 325 (1987); E.J. Eichten and C. Quigg, Phys. Rev. D **49**, 5845 (1994).
 - [3] S. Godfrey and J.L. Rosner, Phys. Rev. D **64**, 097501 (2001); **66**, 059902(E) (and references therein); W. Kwong and J.L. Rosner, *ibid.* **38**, 279 (1988).
 - [4] C.W. Bernard *et al.*, Phys. Rev. D **62**, 034503 (2000); **64**, 054506 (2001).
 - [5] E.J. Eichten *et al.*, Phys. Rev. D **21**, 203 (1980).
 - [6] S. Jacobs, M.G. Olsson, and C. Suchyta III, Phys. Rev. D **33**, 3338 (1986).
 - [7] A.M. Badalian and V.L. Morgunov, Phys. Rev. D **60**, 116008 (1999).
 - [8] E.J. Eichten, K. Lane, and C. Quigg, Phys. Rev. Lett. **89**, 162002 (2002) and references therein.
 - [9] S.J. Brodsky, S. Menke, C. Merino, and J. Rathsmann, Phys. Rev. D **67**, 055008 (2003).
 - [10] A.Yu. Dubin, A.B. Kaidalov, and Yu.A. Simonov, Phys. Lett. B **323**, 41 (1994); Phys. At. Nucl. **56**, 1745 (1993) [Yad. Fiz. **56**, 164 (1993)].
 - [11] Yu.A. Simonov, Phys. At. Nucl. **66**, 764 (2003) [Yad. Fiz. **66**, 796 (2003)]; JETP Lett. **57**, 525 (1993); Phys. At. Nucl. **58**, 107 (1997) [Yad. Fiz. **58**, 113 (1995)]; *Lectures Notes in Physics* (Springer-Verlag, New York, 1996), Vol. 479, p. 138.
 - [12] Yu.A. Simonov, in *Proceedings of the XVII International School of Physics, Lisbon, 1999*, edited by L. S. Ferreira, P. Nogueira, and J. L. Silva-Marcos (World Scientific, River Edge, 2000), p. 60.
 - [13] A.M. Badalian and B.L.G. Bakker, Phys. Rev. D **66**, 034025 (2002).
 - [14] D.P. Stanley and D. Robson, Phys. Rev. D **21**, 3186 (1980); Phys. Rev. Lett. **45**, 235 (1980).
 - [15] J.L. Basdevant and S. Boukraa, Z. Phys. C **28**, 413 (1983); F. Halzen *et al.*, Phys. Rev. D **47**, 3013 (1993); L.P. Fulcher, *ibid.* **44**, 2079 (1991).
 - [16] S. Godfrey and N. Isgur, Phys. Rev. D **32**, 189 (1985).
 - [17] Yu.A. Simonov, Phys. Lett. B **515**, 137 (2001).
 - [18] A.M. Badalian, A.I. Veselov, and B.L.G. Bakker, to be published in Phys. At. Nucl.; hep-ph/03011009.
 - [19] A. Pineda and F.J. Yndurain, Phys. Rev. D **58**, 094922 (1998); B.A. Kniehl *et al.*, hep-ph/0312086 and references therein.
 - [20] A. Billoire, Phys. Lett. **92B**, 343 (1980); M. Peter, Phys. Rev. Lett. **78**, 602 (1997); Y. Schröder, Phys. Lett. B **447**, 321 (1999).
 - [21] Particle Data Group, K. Hagiwara *et al.*, Phys. Rev. D **66**, 010001 (2002).
 - [22] G. Bali, Phys. Lett. B **460**, 170 (1999).
 - [23] A.M. Badalian and D.S. Kuzmenko, Phys. Rev. D **65**, 016004 (2002); A.M. Badalian, Phys. At. Nucl. **63**, 2173 (2001).
 - [24] A.M. Badalian and Yu.A. Simonov, Phys. At. Nucl. **60**, 630 (1997) [Yad. Phys. **60**, 714 (1997)].
 - [25] G. Bali, Phys. Rev. D **62**, 114503 (2000); S. Deldar, *ibid.* **62**, 034509 (2000) and references therein.
 - [26] V.I. Shevchenko and Yu.A. Simonov, Phys. Rev. Lett. **85**, 1811 (2000).
 - [27] G. Parisi and R. Petronzio, Phys. Lett. **94B**, 51 (1980); J.M. Cornwall, Phys. Rev. D **26**, 1453 (1982).